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Visualization Method in Mathematics Classes

Laxmi Rathour[1,*], Dragan Obradovic[2], Shiv Kant Tiwari[3], Lakshmi Narayan Mishra[4], Vishnu Narayan Mishra[5]

[1] Ward Number – 16, Bhagatbandh, Anuppur 484 224, Madhya Pradesh, India; laxmirathour817@gmail.com.
[2] Elementary School "Jovan Cvijic", Kostolac-Pozarevac, Teacher of Mathematics, Serbia; dragishaobradovic@yahoo.com.
[3] Department of Mathematics, L. E. College, Morbi-363 642, Gujarat, India; shivkant.math@gmail.com.
[4] Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore 632 014, Tamil Nadu, India; lakshminarayanmishra04@gmail.com.
[5] Department of Mathematics, Indira Gandhi National Tribal University, Lalpur, Amarkantak 484 887, Madhya Pradesh, India; vishnunarayanmishra@gmail.com.

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Abstract

Developing students' educational abilities has always been one of the most urgent. The more fully a student's potential is realized, the more success a person will be able to achieve in life and career. One of the effective technologies for activating learning is the method of visualizing educational information. The history of visualization in mathematics education is very long. Since the beginning of the year 1980s, mathematics professors have been interested in the practical challenges of teaching visualization, in the visualization of mathematics as an exhibit in school or in accordance with educational psychology, and are looking for theoretical frameworks.

Keywords: Visualization, Mathematical education, Modern education.

1 | Introduction

The method of visualizing educational information is one of the effective technologies of modern education. Visualization allows you to create visual associations, demonstrate the properties of objects and describe the process being studied. This is especially important for mathematics, where the level of abstraction is very high and causes learning difficulties. With the help of visualization, one of the basic principles in teaching mathematics is implemented-the principle of visibility, which allows you to present the teaching material in a more accessible form.

In the process of teaching mathematics, tasks perform different functions. Educational math problems are an effective and often necessary tool for students to master the concepts and methods of the school subject of mathematics and mathematical theory in general. Tasks have a great role in the development of thinking and in the mathematical education of students, in the formation of their

Corresponding Author: laxmirathour817@gmail.com
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skills and abilities in practical applications of mathematics. An effective tool for teaching problem solving is the visualization method. It helps to find the solution path, contributes to a deeper assimilation of solution algorithms, awareness of all the connections present in the problem, helps to see the relationship of concepts, which allows to assess at a higher level their role and importance for the problem in particular and the corresponding theory in general.

2 | Visualization in Mathematics Teaching

The principle of visualization is a basic principle in teaching mathematics and helps in the realization of other didactic principles, because visualization is the external support of mental activities.

Symbolic visualization: The didactic principle of visualization in mathematics teaching is to seek the application of visual means in teaching with the aim of achieving a high degree of abstraction during the formation of mathematical concepts. In the classical methodology of teaching mathematics, two types of visualization are considered.

Subject visualization: Symbolic visualization is achieved with drawings, tables, graphs, schemes, diagrams, graphs of binary relations.

When talking about classical teaching, didactic tools related to this principle are usually prepared in advance and intended for use in the teaching process, as well as text supplements in books and textbooks. This is precisely why the classic methodology deals with the criteria for creating such didactic tools and their application in the teaching process. The connection that such didactic tools must establish with the way of thinking of the student is emphasized, so that he is able to understand exactly what certain drawings, pictures or schemes mean. When, for example, the graph of a real function is displayed, before moving on to studying the properties of functions through the graphical display, the teacher must be absolutely sure that the student can "read" these properties from the graph of functions. Using drawings and graphic representations in teaching, a significant problem is the presentation of three-dimensional, spatial situations. The fact that a three-dimensional situation is presented on a two-dimensional image - on paper or on a blackboard, implies that a projection is used, which for many students is not entirely intuitive and clear. At least in the beginning, until the required connection is established and until the student is able to create stereometric representations on the drawing himself, it is recommended to combine this representation with spatial models.

Subject visualization is achieved by using: Models of plane figures, spatial situations, models of geometric bodies... Again, a model is understood as a physical interpretation of a specific spatial situation, made of any material. In some models, it is possible to see interactions of several objects, for example sections of a geometric body with a plane. With the penetration of information technology in modern teaching, the didactic principle of visualization has gained power many times over. Making accurate and very graphic renderings has become easier and faster. Teachers themselves can create visualization tools associated with each task and each theorem. Also, a large number of such visualizations are easily available, especially via the Internet, and can be implemented in classes. Using networks of teachers, collaborative learning can be achieved, whereby teachers would exchange their didactic materials and make them available to their students, thus transferring the teaching itself from the real classroom in a certain time, to an asynchronous virtual classroom. In the context of symbolic and object visualization, information technologies can be used to create graphics and simulations. Drawings, tables, graphics..., all of that can be produced much faster, even during the teaching process itself. While with classical teaching it was possible to show barely a few graphs of functions during one lesson, with the use of information technology it is possible to do much more and much better. Simulations are more difficult to create, but it is recommended that they be used for exactly the purpose for which physical models are used. The teaching of mathematics in its nature implies the study of concepts, properties, statements and problems that are essentially connected to a certain dynamic process or situation. Starting with the introduction of symbolism in algebra lessons, the student should start thinking dynamically - what happens for this, what for that value of the variable. Each geometry problem asks the student to ask what would happen if one of the geometric objects changed. The study of the functions of the student in particular leads to the constant study of changes. At the very essence of the study of functions is the concept of change, especially when it comes to concepts such as continuity, limit value, derivative and other essential concepts of mathematical analysis.

It is precisely this aspiration in the teaching of mathematics to capture dynamism that makes the application of information technology powerful. In contrast to static multimedia tools (drawings, graphics), dynamic multimedia tools (animations, films, simulations) should be used for this purpose. Such multimedia tools can be used by themselves and integrated into the teaching itself. Multimedia learning systems are a complete didactic tool that can be used for the purpose of realizing the didactic principle of visualization. In recent times, multimedia tools and all other educational materials in electronic format, conforming to appropriate standards, are used as part of more global electronic learning management systems. This raises the very principle of visualization to a higher and more significant level, because now visualization is not only about how to "see" but engages the entire perceptive apparatus of the factors of the teaching process.

3 | The Term, Role and Significance of Mathematical Visualization

The use of visualization in teaching mathematics has its roots in the theory of knowledge and is consistent with the methodology of mathematics. Conventionally, three stages of knowledge can be distinguished: perception, representation and abstract thinking. The process of knowledge can also be conditionally divided into two stages: sensory (perception and representation) and logical (transition from representation to concept with the help of generalization and abstraction). The sensory stage corresponds to the first stage of the path of cognition, and the role of visualization in this stage is quite important. Visualization is used to gain knowledge about the external properties of mathematical objects, about the relationship between objects, about their similarities and differences. The role of visualization in the third level of knowledge lies in the fact that it allows students to show deep connections between the properties of mathematical objects, to create a correct picture.

The role and place of using visual aids in the process of teaching mathematics, as well as the purpose of their use in teaching, depends primarily on the content of the subject and the knowledge the students have. The school should develop a certain range of ideas in students, provide them with the necessary knowledge and skills and teach them how to apply the acquired knowledge in practice. It is necessary to create an atmosphere in the classroom in which students would be interested in mathematics, to awaken in students the desire to learn mathematics. The use of visualization in the classroom facilitates the perception and understanding of educational material by students, helps to develop interest in mathematics, and also to connect theoretical information more closely with practice. The method of visual teaching of mathematics plays a significant role in the difficult struggle against the formalism of school knowledge and its isolation from life practice.

Visualization is an important aspect of mathematics, mathematical thinking, understanding and reasoning. The role of visualization in the learning process and its impact on educational results are the subject of many studies conducted so far. The authors point out that visual thinking is very important for the learning process, they present convincing arguments about the central role of visualization in the reform changes of mathematics teaching [1], [2], [4], but they also indicate the possible difficulties and limitations of visualization [9], [13]-[15]. In her paper, Presmeg [9] provided an overview of previously conducted research related to visualization in mathematics learning and teaching. She points out that more active dealing with this topic began in the nineties of the last century, more precisely, after the Psychology of Mathematics Education (PME)-13 conference, where several papers on visual representations and visual thinking were exhibited. Presmeg notes that in the existing literature, theterm "visualization" has different meanings and gives his view of this phenomenon. Visualization includes the processes of constructing and transforming both visual mental images and all sketches of a spatial nature that may be related to doing mathematics. With the development of computers and their graphic capabilities, visualization gained more and more importance and topicality. To date, a large number of works have been published dealing with visualization, visual thinking, visual representations, visual methods of problem solving, and visual approaches to teaching/learning, etc.

The concept of visual thinking, which is widely used today, was introduced by Arnheim [8], defining it as thinking through visual operations. In visualization, images combine aspects of natural representation with more formal forms and thereby increase cognitive understanding. Arnheim points out that visual images are not only an illustration of thought, but much more, they are the development of thought itself. The title of his work is a very concise definition of visual thinking: "visual thinking - Unity of image and concept" which at the same time gives a significant implication for the teaching of mathematics. A very comprehensive definition of visualization was given by Arcavi [2]: "Visualization is the ability, process and product of creating, interpreting and using images, diagrams, illustrations in our mind, on paper or with technological means, the purpose of which is to display and communicate information, think and develop previously unknown ideas and advancing understanding" [2]. This definition highlights the importance of visualization for mathematics teaching/learning in the following aspects:

I. Visual representations (both external and internal) are important for presenting mathematical ideas and developing students' abilities and skills in mathematical communication.
II. Visualization can give clearer meaning and meaning to mathematical concepts and connections between them and thereby contribute to the development of conceptual understanding.
III. Visualization can be a powerful tool for exploring and solving mathematical problems.

Visual mathematical representations can be defined as a set of graphic symbols that visually encode the causal, functional, structural and semantic properties and relationships of the mathematical concepts they represent [10]. In teaching mathematics, geometry is the most natural area in which visual representations are used. However, visual representations can also be used in other areas of mathematics to encourage visual thinking. Visual representations are present in the contents of trigonometry, analytical geometry, real functions, differential and integral calculus, but they are insufficiently represented in teaching. When processing some concepts (eg the first derivative of a function), graphical representations are used when introducing the concept, and after that, they are often neglected and the work is dominated by algebraic representations. Visualization is a central component of many processes in which the transition from a concrete to an abstract model of thinking is made [11]. Insufficient representation of visual representations, as carriers of the meaning of concepts, can lead to premature formalization. In such situations, students rely on formulas and ready-made algorithms and carry out algebraic procedures, without a clear understanding of the meaning. The role of visualization in solving mathematical problems is very significant. Visual representations present information clearly and can serve to reveal the structure of a problem and lay the foundation for its solution [7]. If algebraic and visual approaches to problem solving are compared, it can be concluded that visual approaches have certain advantages: 1) Visualization allows reducing complexity when working with a lot of information, so the task can be solved more simply and quickly, 2) Visualization can contribute to solving cognitive conflicts between (correct) symbolic and (incorrect) intuitive solutions [2], 3) Visual approaches can point to the conceptual foundations of problems that students can easily ignore in algebraic procedures [2], and 4) The concreteness of visual representations is an important factor for creating the effect of immediacy in proof [6]. Some formal proofs can be replaced by geometrically analogous ones, simple and beautiful so that the validity of the theorem almost blinds us [5].

Visual representations represent a combination of concrete and abstract elements of the mathematical structure of the problem. Therefore, they can bridge the gap between the concrete and abstract side of the problem and can facilitate both the mathematization and concretization of the problem. The representation of concrete/abstract elements determines the degree of concreteness (or abstractness) of the visual representation. Concrete visual representations are directly related to the real situation, they are easy to understand, and they are interesting and can increase the motivation of students. However, they do not provide enough information that clearly shows the mathematical relationships and structure

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of the problem, that is, the connection with the abstract side of the problem is weak. Abstract visual representations are characterized by the use of conventional graphic symbols to represent relevant structural elements. They help students focus on the essential features of the problem, but are more difficult to understand and may require better prior knowledge. In relation to concrete, abstract representations are "more powerful", because they can overcome the context in the reasoning and problem-solving process [3]. If concrete and abstract visual representations are simultaneously used to visualize the problem, the advantages of both representations can be used. In this way, students are facilitated to establish relationships and connections between concrete and abstract aspects of the problem, which ultimately leads to further improvement of the learning process and better student achievements [12]. By introducing visual approaches in the teaching of mathematics, students are given the opportunity to acquire comprehensive and functional knowledge. The use of visual representations should be designed and well measured, so as not to lead to the other extreme. Visualization takes on its true, essential meaning only if visual representations are connected to algebraic, numerical, and verbal representations [1].

The problem of ability is a problem of individual differences. With the best organization of teaching methods, the student will progress more successfully and faster in one area than in another. Of course, success in learning is not determined only by the student's abilities. In this sense, the content and teaching methods, as well as the students' attitude towards the subject, are of primary importance. Therefore, success and failure in learning do not always provide a basis for judgments about the nature of the student's abilities.

The presence of weak abilities in students does not free the teacher, as far as possible, to develop the abilities of these students in this area. At the same time, there is an equally important task - to fully develop his abilities in the area in which he shows them. It is necessary to educate and select the capable ones, not forgetting all schoolchildren, to raise their general level of competence in every possible way. In this regard, different collective and individual work methods are needed in their work in order to activate the students' activity in this way.

The learning process should be comprehensive both in terms of the organization of the learning process itself, and in terms of developing students' deep interest in mathematics, skills and abilities to solve tasks, understanding the system of mathematical knowledge, solving a special system of non-standard tasks with students, which should be offered not only in lessons, but also on tests. Therefore, a special organization of teaching material presentation, a well-designed task system, contribute to increasing the role of meaningful motives for learning mathematics. The number of result-oriented students is decreasing. In the class, not only problem solving should be encouraged in every possible way, but also the unusual way of solving problems that students use, whereby special importance is attached not only to the result in solving the problem, but also to the beauty and rationality of the method.

Teachers successfully use the technique of "setting tasks" to determine the direction of motivation. Each task is evaluated according to a system of the following indicators: the nature of the task, its correctness and relation to the original text. The same method is sometimes used in the wine version: after solving the task, students were asked to put all the problems together in some way related to the original problem. To create psycho-pedagogical conditions for increasing the effectiveness of the organization of the learning process system, the principle of organizing the learning process in the form of subject communication using cooperative forms of student work is used. This is group problem solving and collective discussion on assessment, pair work and team work.

4 | Conclusion

Visualization in teaching is not only about how to "see". It should engage the entire perceptive apparatus of the student as a factor in the teaching process. That is why the didactic principle of visualization is one of the foundations of modern mathematics teaching. The didactic principle of visualization in mathematics teaching is to seek the application of visual means in teaching with the aim of achieving a high degree of abstraction during the formation of mathematical concepts.

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In order to comprehensively understand and solve problems related to teaching and learning mathematics, mathematics education researchers pay significant attention to mathematical representations, visualization and modern educational technologies, their role and importance for the learning process. Multiple representations, visualization and educational technology, recognized as necessary components of mathematics education, need to be implemented in all segments of teaching, due to their potential to promote mathematical insight and understanding and improve the learning process. The mentioned aspects are particularly important for the study of functions, a fundamental concept of mathematics teaching.

Visual learning allows you to organically connect the abstract with the concrete, combine scientific and accessible presentation of materials, individual and collective forms of work, solve problems of a developmental nature, creatively and proactively approach the teacher's work, consider individual and age characteristics of perception, memory and thinking, to develop students' interest and cognitive abilities through skillful organization of work on designing visual aids.

Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there are no conflicts of interest to report. We certify that the submission is original work and is not under review at any other publication.

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